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On the Interaction between Aggregators, Electricity Markets and Residential Demand Response Providers

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***Abstract*—To decarbonize the heating sector, residential consumers may install heat pumps. Coupled with heating loads with high thermal inertia, these thermostatically controlled loads may provide a significant source of demand side flexibility. Since the capacity of residential consumers is typically insufficient to take part in the day-ahead electricity market (DAM), aggregators act as mediators that monetize the flexibility of these loads through demand response (DR). In this paper, we study the strategic interactions between an aggregator, its consumers and the DAM using a bilevel optimization framework. The aggregator-consumer interaction is captured either as a Stackelberg or a Nash Bargaining Game, leveraging chance-constrained programming to model limited controllability of residential DR loads. The aggregator takes strategic positions in the DAM, considering the uncertainty on the market outcome, represented as a stochastic Stackelberg Game. Results show that the DR provider-aggregator cooperation may yield significant monetary benefits. The aggregator cost-effectively manages the uncertainty on the DAM outcome and the limited controllability of its consumers. The presented methodology may be used to assess the value of DR in a deregulated power system or may be directly integrated in the daily routine of DR aggregators.**

***Index Terms*—Aggregator, Chance-constrained Programming, Nash Bargaining Game, Stackelberg Game, Demand Response, Thermostatically Controlled Loads**

NOMENCLATURE

Below, we list all sets, parameters, primal variables and functions used in Section II-C. Dual variables are listed after a colon in each constraint and are not listed here.

A. Sets

$\mathcal{H}$	Set of consumers, indexed by h.
$\mathcal{I}$	Set of generators, indexed by i.
S^{A}, S^{H}_h	Set of cooperation strategies of the aggregator (A) or consumer h (H).
$S^{\mathrm{A}*}$, $S^{\mathrm{H}*}_h$	Set of disagreement strategies of the aggregator (A) or consumer h (H).
$\mathcal{T}$	Set of time periods, indexed by t.
Ω	Set of RES scenarios, indexed by ω.

B. Primal variables

$\mathcal{B}^{\mathrm{A}}$	Benefit associated with the aggregator - consumer cooperation, €.
$\mathcal{B}^{\mathrm{A}}$, $\mathcal{B}^{\mathrm{H}}_h$	Benefit associated with the cooperation strategy for the aggregator (A) or consumer h (H), €.
$\mathcal{B}^{\mathrm{A}*}$, $\mathcal{B}^{\mathrm{H}*}_h$	Benefit associated with the disagreement strategy for the aggregator (A) or consumer h (H), €.
$d_{t,\omega}$	Electricity purchased by demand at time period t in scenario ω, MWh.
$d^{\mathrm{H}}_{h,t}$	Electricity demand of consumer h at time period t, MWh.
D^{H}_t	Stochastic electricity demand of all consumers at time period t, MWh.
$g_{i,t,\omega}$	Electricity sold by generator i at time period t in scenario ω, MWh.
$\lambda^{\mathrm{A}}_{h,t}$	Retail electricity price for consumer h at time period t, €/MWh.
ψ_t	Auxiliary variable.
$q^{\mathrm{agg}}_{t,\omega}$	Electricity purchased by the aggregator at time period t in scenario ω, MWh.
$\overline{Q^{\mathrm{agg}}_t}$	Aggregator's bidding quantity at time period t, MWh.
R^{A}_h, R^{R}_h	Aggregator (A) or retailer (R) revenue from consumer h, €.
$\boldsymbol{\theta_{h,t}}$	Temperature in building h at time step t, K.
$w_{t,\omega}$	Electricity sold by non-controllable renewable sources at time period t in scenario ω, MWh.
x^{A}, x^{H}_h	Division factors governing the split of the benefit $\mathcal{B}$ between the aggregator (A) or consumer h (H).

C. Parameters

A_h	State-space model matrix.
C_h	Coefficient of performance of heat pump h.
δ^{P}, δ^{NP}	Normally distributed disturbance, i.e., $\delta \sim N(0,\sigma)$, proportional (P) or non-proportional (NP), - (P) or MW (NP).
$\underline{\Delta R^{\mathrm{R}}_h}$	Lower limit to the decrease in consumer h's electricity bill, €.
$\underline{\Delta R^{\mathrm{A}}_h}$	Lower limit to the aggregator's profit per consumer h, €.
D_t	Capacity of demand during time period t, MW.
ϵ	Characterization of the aggregator's risk attitude.

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$E_{h,t}$	Disturbance in building h at time step t, K.
$\overline{G_i}$	Capacity of generator i, MW.
N	Total number of consumers.
NB_h	Number of buildings of category h.
M	Arbitrary large constant.
π_ω	Probability of occurrence of scenario ω.
P^{agg}	Aggregator's bidding price, €/MWh.
P_i^{g}	Offering price of generator i, €/MWh.
P^{d}	Bidding price of demand, €/MWh.
$\overline{P_h}$	Capacity of heat pump and auxiliary heater h, kW.
$\overline{\boldsymbol{\theta_{h,t}}}$	Upper temperature bound set by consumer h at time period t, K.
$\underline{\boldsymbol{\theta_{h,t}}}$	Lower temperature bound set by consumer h at time period t, K.
$\overline{W_{t,\omega}}$	Available output of non-controllable renewable sources at time period t in scenario ω, MWh.
y^{A}, y_h^{H}	Bargaining power of the aggregator (A) or consumer h (H).

D. Functions

Φ^{-1}	Inverse cumulative probability density function of the standard Normal distribution.
$\mathcal{G}$	Transfer function describing the linear thermal model, governing the relation between electric heating demand and temperatures.

I. Introduction

Residential demand response (DR) resources, such as thermostatically controlled loads (TCLs), may offer the flexibility to cost-effectively integrate intermittent electricity generation from renewable energy sources (RES). The whole-system value of these distributed DR resources has been extensively studied (e.g., [1]–[3]). This value has been recognized by the European Commission, which encourages Member States to open electricity markets for DR resources [4]. To bring this small-scale flexibility to large-scale markets, aggregators may be required. System-level studies, however, typically do not consider these market participants.

Therefore, we study the strategic interactions between a DR aggregator, DR providers and a day-ahead electricity market (DAM). This research is motivated by the emergence of DR aggregators, leveraging residential TCLs to provide system-level services and/or to participate in electricity markets (e.g., [5]–[7]). Furthermore, we consider the transfer of risk from the DR providers - whose loads may be limitedly controllable - to the aggregator. By pooling loads and their controllability characteristics, the risk of the overall portfolio may be easier to manage, allowing more profitable, less risk-averse positions in the market without sacrificing the reliability of the end-energy service for the DR provider.

Two distinct perspectives may be adopted to study the interaction between a DR aggregator and an electricity market. First, one may study the aggregator as a price-taking agent via optimization models, e.g. [8], [9]. Xu et al. [8] study a risk-averse aggregator of distributed generation and electric vehicles. Mathieu et al. [9] calculate an upper bound to the profit a price-taking aggregator of TCLs may attain via arbitrage in the intraday electricity market. Alternatively, a bilevel optimization problem may be used to represent the relation between a strategic, price-making aggregator and the market clearing outcome, reflecting a Stackelberg Game [10]–[13]. Kardakos et al. [10] develop optimal bidding strategies for a virtual power plant consisting of, i.a., DR loads. Similarly, Nekouei et al. [11] study a strategic aggregator, offering load reduction, competing with a set of generating companies in a market environment. Kazempour et al. [12] optimize bidding curves for a single large consumer under uncertainty on the offers and demand bids of other non-strategic agents. In addition, Ruhi et al. [13] examine opportunities for price manipulation by aggregators through strategic curtailment of generation in a market environment.

Similarly, the relationship between an aggregator and its DR providers may be represented as a Stackelberg game [14]–[18]. Zugno et al. [14] study the profitability of a retailer with DR consumers in a joint energy-reserve market. Li et al. [15] study an aggregator maximizing the social welfare by coordinating a group of TCLs, subject to a peak demand constraint, by defining proper retail price signals. Yu and Hong [16] formulate a single-leader-multiple-follower Stackelberg game to determine optimal control signals for DR loads. Similarly, Neyestani et al. [17] study the interaction between, i.a., an electric vehicle parking lot and an aggregator via a Stackelberg Game. Yazdani-Damavandi et al. [18] develop a single-leader-multiple-follower Stackelberg Game representing the interaction between an aggregator serving the natural gas, heat and electricity demand of local energy communities and the DAM. Alternatively, several authors have introduced Nash Bargaining theory [19], [20] to mimic the long-term consumer-aggregator cooperation [21]–[25]. Contreras et al. [21] study the repeated interaction between an aggregator, strategically participating in the DAM, and distributed, perfectly controllable energy storage systems. Hoa et al. [22] use a Nash Bargaining Game to study the allocation of power/energy to a number of aggregators managing, i.a., TCLs , represented via virtual battery models. Ye et al. [23] study the redistribution of the benefits of sharing energy storage systems and distributed generation capacity in a residential community. Given a desired load reduction, Guo et al. [24] use a Nash Bargaining Game to determine the load reduction of each tenant in a colocation data center and the reimbursement offered to these tenants by the data center operator. Nguyen et al. [25] model the interaction between distributed generation resources offering reactive power and the electric utility company using Nash Bargaining Theory.

Typical residential DR models, however, simplify the complex interactions between the supply and demand side, which may lead to erroneous estimates of the value of DR resources [26]. Although some researchers study the impact of an uncertain availability of DR resources [27], they customarily assume perfect control, i.e., the ability to externally adjust the state of these loads. In practice, DR providers are unlikely to perfectly match aggregators' expectations [28].

In this paper, we formulate a bilevel optimization problem mimicking the strategic participation of a price-making aggre-

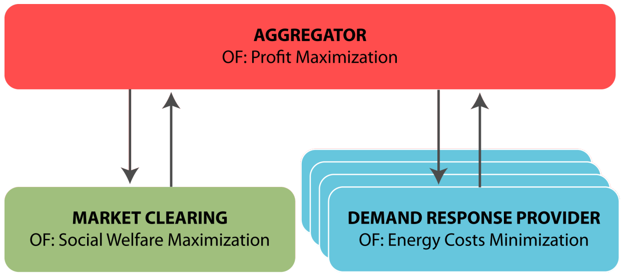

Fig. 1. Interaction between the aggregator, the market and the demand response providers and their objective functions (OF) [29].

gator in the DAM (Stackelberg Game) and the interaction of said aggregator with its limitedly controllable DR providers (Stackelberg or Nash Bargaining Game). Leveraging a detailed residential heating system and load model [26], chance-constrained programming [3] and game theory [19], [20], this modeling framework allows studying the participation of the DR aggregator in a DAM while guaranteeing that all user-defined comfort constraints are met. Our contribution w.r.t. the scientific literature and our conference paper [29] is twofold:

1) The interaction between the aggregator and its DR providers is represented as a Stackelberg and a Nash Bargaining Game. In absence of explicit constraints on the formation of the retail price, the set of solutions of the Stackelberg Game is shown to be a subset of the set of solutions of the Nash Bargaining Game. The Nash Bargaining Game allows representing the DR providers and the aggregator as a single entity participating in the DAM, simplifying the required model. The consumer-aggregator Stackelberg Game and the relation of its solutions with those obtained under the assumption of a Nash Bargaining Game are not considered in [29].

2) To determine the optimal risk attitude of the aggregator, we analyze the impact of different bidding strategies and DR load realizations on the aggregator's profitability via out-of-sample day-ahead and intra-day market simulations. By considering mark-ups on intra-day market prices, we mimic possible risk premiums and limited liquidity in these markets. The inherent uncertainty on the DAM outcomes (scenarios) and limited controllability of DR providers (chance-constraints) is explicitly considered during the optimization of the aggregator's bids. The chance constraints are analytically reformulated using convex second-order conic constraints, as in [3], which preserves computational resources for solving an NP-hard, stochastic problem. In [29], we did not consider the uncertainty on the DAM outcome, nor did we perform out-of-sample simulations to determine the aggregator's optimal risk policy.

Regulators, policy makers and power system operators may use the method described below to assess the value of DR in a deregulated power system, which may inform a detailed cost-benefit analysis of the deployment of DR infrastructure. Aggregators may integrate this framework with their daily routines to account for the limited controllability of the DR resources and their own risk attitude.

The remainder of this paper is structured as follows. Section II-A introduces the agents and the interactions between them. The mathematical formulation is presented in Section II-C. A case study, inspired on the Belgian power system, is laid out in Section III. Section IV provides some concluding remarks.

II. Methodology

First, we provide a brief description of the agents (the aggregator, the DR providers and the market operator), their objectives and the interactions between the agents. Second, we introduce an example to conceptually illustrate the Nash Bargaining and Stackelberg Game between the aggregator and the DR providers. Last, the mathematical formulation of the optimization problem solved by each agent and their dependencies are discussed in detail.

A. Description of the Game

1) Agents: We consider three types of agents: the aggregator, the DR providers and the market operator (Fig. 1). The market operator maximizes social welfare while ensuring the demand for electricity is met by conventional and renewable generation. The DR providers each minimize their energy cost associated with space heating and domestic hot water consumption, which is governed by user-specified comfort constraints. The aggregator aims to maximize its operating profit. Its revenue depends on the consumption of the DR providers and the retail price it charges the DR providers. The expenses of the aggregator depend on the DAM clearing price, which is uncertain due to uncertainty on the available RES-based generation and is determined endogenously. The aggregator is risk-neutral w.r.t. uncertainty on market clearing prices, i.e., it minimizes its expected procurement cost.

2) Interactions between agents: The aggregator (leader) decides on its bid in the DAM (follower) and the price charged to the DR providers (followers) based on its expectation of the DAM clearing and the demand of its DR providers. The uncertainty in the aggregators expectation of the market clearing (price), which stems from wind power forecast errors, is represented via scenario-based stochastic programming, whereas we leverage chance-constrained programming to reflect the limited controllability of DR loads, as in [3]. The notion of limited controllability refers to possible real-time deviations from an expected DR load profile, e.g., due to sub-rational consumer behavior or an incomplete DR load model employed by the aggregator. In contrast to the risk-neutral attitude of the aggregator w.r.t. the DAM prices, we assume a risk-averse attitude w.r.t. the possible real-time deviations from the expected DR load, which is reflected in the chance constraints. These constraints ensure that the aggregator procures sufficient energy in the DAM to cover the DR load in a predefined percentage of all possible load realizations. The 'risk' that the aggregator tolerates defines the amount of energy that needs to be procured in intraday, real-time or balancing markets. Note that the associated intraday procurement cost is not explicitly considered in the aggregator's day-ahead bid problem. By adopting this approach based on chance constraints, however, we avoid a scenario-based representation of the intra-day electricity market (IDM) in the day-ahead decision problem of the aggregator. This would require considering a set of IDM scenarios (representing the aggregator's position in each heating demand realization) *for each DAM scenario*. Hence, the required number of scenarios may quickly explode, which may lead to computational intractability.

The interaction of the aggregator and the DR providers may take three distinct forms, which we will refer to as 'Retailer', 'Nash Bargaining Game' and 'Stackelberg Game'. In the retailer paradigm, the aggregator provides a flat rate to its consumers, which removes all incentives to utilize their flexibility. If the aggregator and the DR providers cooperate, the game governing the division of the benefits of this interaction may be represented as a Nash Bargaining Game, explicitly accounting for power relation between the aggregator and the DR providers. If the aggregator unilaterally decides on a time-varying rate for the consumer while anticipating its rational reaction, the interaction between the aggregator and the consumers may be represented as a Stackelberg game.

3) Assumptions: The aggregator is the only strategic price-maker in the DAM. Since we consider day-to-day operations, DR providers may not switch between aggregators or retailers. This assumption is motivated by the low switching rates observed in European retail markets [30]. In the optimization of the aggregator's bids, we only consider the DAM, neglecting arbitrage opportunities with intra-day and balancing markets. Transmission constraints are not considered in the market clearing problem, as in [3], [31]. Although this assumption masks the location-specific value of DR in congestion-prone power systems, it will have a limited impact on the results if transmission constraints are not binding on a regular basis. The Belgian power system that provides the basis for our simulations has enough (internal) transmission capacity to make the effect of congestion essentially negligible [32]. Furthermore, from a market perspective, the Belgian day-ahead market is cleared as one zonal market (with a single price). Transmission constraints are checked in a second stage by the transmission system operator with potential redispatching to alleviate congestion [33]. The only sources of uncertainty in the DAM are imperfect wind power forecasts. Demand response providers may be limitedly controllable. The possible deviations from their expected demand are represented via normal distributions (see Eq. (10), which is based on [3]).

B. Illustrative example: Nash Bargaining vs. Stackelberg

Consider a two-period DAM, for which the clearing prices are known in advance: $\lambda_t = \{20, 40\}$ (€/MWh) with $t \in \{1,2\}$. A single, perfectly controllable DR provider has a baseline consumption of 1 MWh in each time step: $d_t^{\mathrm{H}} = \{1,1\}$. Hence, a profit-neutral retailer will charge the consumer $R^{\mathrm{R}} = 60$ € or $\lambda^{\mathrm{R}} = 30$ €/MWh to recover its expenses in the wholesale market.

If this consumer (follower) would engage in a Stackelberg Game with an aggregator (leader), we may formalize this as:

$$\text{Max.} \sum_{t\in 1,2} \lambda_t^{\mathrm{A}} \cdot d_t^{\mathrm{H}} - \lambda_t \cdot d_t^{\mathrm{H}} \tag{1}$$

subject to

$$\sum_{t\in 1,2} \lambda_t^{\mathrm{A}} \cdot d_t^{\mathrm{H}} \leq R^{\mathrm{R}} \tag{2}$$

$$d_t^{\mathrm{H}} = \text{argmin}\{\sum_{t\in\{1,2\}} \lambda_t^{\mathrm{A}} \cdot d_t^{\mathrm{H}} \text{ s.t. } d_1^{\mathrm{H}} + d_2^{\mathrm{H}} = 2, \;\; d_1^{\mathrm{H}}, d_2^{\mathrm{H}} \geq 0\} \tag{3}$$

The aggregator maximizes the difference between what it charges to the consumer $\sum_{t\in 1,2} \lambda_t^{\mathrm{A}} \cdot d_t^{\mathrm{H}}$ and its expenses on the wholesale market $\sum_{t\in 1,2} \lambda_t \cdot d_t^{\mathrm{H}}$. Constraint (2) ensures the cost for the consumer is limited to that under the retailer paradigm. Problem (3) represents the cost minimization problem of a rational consumer. The aggregator will maximize its profit by shifting all demand to the first time step. The retail rate in the first time step is only limited by Eq. (2), which will always be binding. Hence, $\lambda_t^{\mathrm{A}} = \{30, \overline{\lambda}\}$, with $\overline{\lambda}$ the retail price cap, and $\sum_{t\in 1,2} \lambda_t^{\mathrm{A}} \cdot d_t^{\mathrm{H}} = R^{\mathrm{R}}$.

Assuming a Nash Bargaining Game between the aggregator and the consumer, they may be represented as a single entity participating in the market [21]. The Nash Bargaining Game determines how the benefit of this collaboration $\mathcal{B}$, i.e., the maximum attainable cost savings for the consumer or maximum profit for the aggregator and defined as $R^{\mathrm{R}} - \sum_{t\in 1,2} \lambda_t \cdot d_t^{\mathrm{H}}$, is split between the aggregator and the consumer[1]:

$$\text{Max.} \; (x^{\mathrm{A}} \cdot \mathcal{B})^{y^{\mathrm{A}}} \cdot (x^{\mathrm{C}} \cdot \mathcal{B})^{y^{\mathrm{C}}} \tag{4}$$

subject to

$$x^{\mathrm{A}} + x^{\mathrm{C}} = 1, \;\; y^{\mathrm{A}} + y^{\mathrm{C}} = 1, \;\; x^{\mathrm{A}}, x^{\mathrm{C}}, y^{\mathrm{A}}, y^{\mathrm{C}} \geq 0 \tag{5}$$

$$d_1^{\mathrm{H}} + d_2^{\mathrm{H}} = 2, \;\; d_1^{\mathrm{H}}, d_2^{\mathrm{H}} \geq 0 \tag{6}$$

with x^{A} and x^{C} the relative share of the benefit the aggregator and the consumer claim. y^{A} and y^{C} represent their bargaining power. Assuming the aggregator has all bargaining power ($y^{\mathrm{A}} = 1$, $y^{\mathrm{C}} = 0$) allows reformulating Objective (4) as

$$\text{Max.} \; x^{\mathrm{A}} \cdot (R^{\mathrm{R}} - \sum_{t\in 1,2} \lambda_t \cdot d_t^{\mathrm{H}}) \tag{7}$$

In this specific case, the aggregator may claim all the benefits of the collaboration ($x^{\mathrm{A}} = 1$), as in the Stackelberg Game. The remaining problem, describing the day-to-day decision problem faced by the aggregator, is identical to problem (1)-(3). Hence, the set of solutions of problem (1)-(3) (Stackelberg Game) is, if it exists, enclosed in the set of solutions to problem (4)-(6) (Nash Bargaining Game).

C. Mathematical Formulation

First, the optimization problems faced by each of the agents are introduced (Sections II-C1, II-C2 and II-C3). Second, we describe the interaction between the aggregator and the market operator (Section II-C4) and the aggregator and the DR providers (Section II-C5). Third, the resulting mathematical problems with equilibrium constraints (MPECs) are reformulated (using KKT conditions, the strong duality theorem and linearization techniques) as equivalent mixed integer quadratically constrained programming (MIQCP) problems in Section II-C6.

1) Aggregator's Perspective: The aggregator aims to maximize its profit, which is the difference between the revenue it obtains from its consumers and the cost of procuring electricity in the DAM (Eq. (8)):

$$\text{Max.} \sum_{t\in\mathcal{T}} \left[\sum_{h\in\mathcal{H}} NB_h \cdot R_h^{\mathrm{A}}(\lambda_{h,t}^{\mathrm{A}}, d_{h,t}^{\mathrm{H}}) - \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q_{t,\omega}^{\mathrm{agg}} \right] \tag{8}$$

[1] We introduce the Nash Bargaining Game without formalizing the associated cooperation and disagreement strategies. For the specific consumer-aggregator problem at hand, the Nash Bargaining Game is formalized in Section II-C5c. For a more general background, we refer the interested reader to Osborne and Rubinstein [20].

subject to

$$\mathbb{P}\big(\overline{Q_t^{\text{agg}}} \geq D_t^{\text{H}},\ \forall t \in \mathcal{T}\big) \geq 1-\epsilon \tag{9}$$

The aggregrator charges consumers a retail rate $\lambda_{h,t}^{\text{A}}$, resulting in a revenue $\sum_{h\in\mathcal{H}} NB_h \cdot R_h^{\text{A}}(\lambda_{h,t}^{\text{A}}, d_{h,t}^{\text{H}})$, required to cover its expected expenses in the DAM $\sum_{\omega\in\Omega}\pi_\omega \cdot \lambda_{t,\omega} \cdot q_{t,\omega}^{\text{agg}}$. Note that the retail tariff $\lambda_{h,t}^{\text{A}}$ and the revenue $R_h^{\text{A}}(\lambda_{h,t}^{\text{A}}, d_{h,t}^{\text{H}})$ depend on the mechanism governing the interaction between the DR providers and the aggregator (Section II-C5). Likewise, the interaction between the aggregator and the market clearing will determine the DAM price $\lambda_{t,\omega}$, hence the aggregator's expenses (Section II-C4). Chance constraint (9) forces the aggregator to procure sufficient electricity in the DAM to cover the demand of its consumers D_t^{H} at each time step with a probability of $(1-\epsilon)\cdot 100\%$. If the energy procured in the DAM does not suffice to meet the real-time heating load, the aggregator may procure additional energy in intraday and real-time markets, possibly at a premium (see Section III). Parameter ϵ can be interpreted as the risk-attitude of the aggregator, following [34]. Smaller values of ϵ reflect a more risk-averse aggregator, whereas setting ϵ equal to 0.5 is a strategy pursued by a risk-neutral aggregator. Recall that the risk-attitude of the aggregator only relates to the volume procured in the DAM (Section II-A2).

In this paper, we assume that this limitedly controllable, stochastic demand can be characterized by a proportional (δ^{P}) and a non-proportional (δ^{NP}) deviation – designed to reflect the possible limited controllability of the DR load – from an expected demand profile $\sum_{h\in H} NB_h \cdot d_{h,t}^{\text{H}}$, with NB_h as scale factor, indicating the number of consumers of type h, and $d_{h,t}^{\text{H}}$ governed by Eqs. (19)–(21):

$$D_t^{\text{H}} = (1+\delta^{\text{P}}) \cdot \sum_{h\in H} NB_h \cdot d_{h,t}^{\text{H}} + \delta^{\text{NP}} \tag{10}$$

This representation of the stochastic demand D_t^{H} stems from [3], in which we analyzed the difference between the expected and actual demand of a set of controllable heat pumps. This analysis revealed a weak correlation between this difference and the expected demand, which suggested that stochastic variable D_t^{H} can be characterized by a proportional (δ^{P}) and a non-proportional (δ^{NP}) disturbance to the expected, aggregated demand profile $\sum_{h\in H} NB_h \cdot d_{h,t}^{\text{H}}$. Note that δ^{NP} is an absolute term, expressed in MW, whereas δ^{P} is a relative term expressed as a percentage of the expected demand.

The disturbances δ^{P} and δ^{NP} are assumed to follow a Normal distribution, i.e., $\delta^{\text{P}} \sim N(0, (\sigma^{\text{P}})^2)$ and $\delta^{\text{NP}} \sim N(0, (\sigma^{\text{NP}})^2)$, as in [3]. Under these assumptions, chance constraint (9) can be analytically recast as [3], [34]

$$\overline{Q_t^{\text{agg}}} = \sum_{h\in H} NB_h \cdot d_{h,t}^{\text{H}} + \Phi^{-1}(1-\epsilon) \cdot (\psi_t + \sigma^{\text{NP}}),\ \forall t \in \mathcal{T} \tag{11}$$

$$\psi_t^2 \geq \big(\sigma^{\text{P}} \cdot \sum_{h\in H} NB_h \cdot d_{h,t}^{\text{H}}\big)^2,\ \forall t \in \mathcal{T} \tag{12}$$

where ψ_t is an auxiliary decision variable, Φ^{-1} is the inverse cumulative probability density function of the standard Normal distribution and Eq. (12) is a second order conic constraint. The decision variables of the aggregator are the retail tariff $\lambda_{h,t}^{\text{A}}$ and the demand bid $\overline{Q_t^{\text{agg}}}$. The risk attitude of the aggregator, reflected by parameter ϵ, is assumed to be known and exogenous to the model. For more information on the reformulation, interpretation and application of chance constraints, we refer the interested reader to Bienstock et al. [34].

2) Market Operator's Perspective: The market operator aims to maximize the total surplus with respect to the bids and offers of the market participants. We assume that the only source of uncertainty (from the perspective of the aggregator) is the available RES-based generation, as offered by other market participants. For each scenario ω of the available RES-based generation $\overline{W_{t,\omega}}$, the problem is formulated as follows, based on Pandžić et al. [35] (dual variables of each constraint are listed after a colon):

$$\text{Max.} \sum_{t\in\mathcal{T}} \big[P^{\text{d}} \cdot d_{t,\omega} + P^{\text{agg}} \cdot q_{t,\omega}^{\text{agg}} - \sum_{i\in\mathcal{I}} P_i^{\text{g}} \cdot g_{i,t,\omega}\big] \tag{13}$$

subject to:

$$-w_{t,\omega} - \sum_{i\in\mathcal{I}} g_{i,t,\omega} + d_{t,\omega} + q_{t,\omega}^{\text{agg}} = 0, \quad \forall t \in \mathcal{T} \quad : \lambda_{t,\omega} \tag{14}$$

$$0 \leq g_{i,t,\omega} \leq \overline{G_i}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad : \underline{\delta_{i,t,\omega}}, \overline{\delta_{i,t,\omega}} \tag{15}$$

$$0 \leq d_{t,\omega} \leq \overline{D_t}, \quad \forall t \in \mathcal{T} \quad : \underline{\epsilon_{t,\omega}}, \overline{\epsilon_{t,\omega}} \tag{16}$$

$$0 \leq w_{t,\omega} \leq \overline{W_{t,\omega}}, \quad \forall t \in \mathcal{T} \quad : \underline{\zeta_{t,\omega}}, \overline{\zeta_{t,\omega}} \tag{17}$$

$$0 \leq q_{t,\omega}^{\text{agg}} \leq \overline{Q_t^{\text{agg}}}, \quad \forall t \in \mathcal{T} \quad : \underline{\kappa_{t,\omega}}, \overline{\kappa_{t,\omega}} \tag{18}$$

Conventional demand and the aggregator bid into the market at P^{d} and P^{agg}. Conventional generation is offered at average generation cost P_{i}^{g}. Renewable generation is offered at 0 €/MWh to ensure it is accepted. Constraint (14) is the power balance equation and includes injections of RES-based generation $w_{t,\omega}$ and conventional generators $g_{i,t,\omega}$, as well as demand $d_{t,\omega}$ and aggregator $q_{t,\omega}^{\text{agg}}$ quantities. Constraint (15) ensures a generator i (set $\mathcal{I}$) cannot sell electricity above its capacity $\overline{G_i}$, while Constraint (16) limits the electricity purchased by the demand to $\overline{D_t}$. Constraint (17) limits the electricity sold by non-controllable renewable sources to their available generation $\overline{W_{t,\omega}}$. Electricity purchased by the aggregator $q_{t,\omega}^{\text{agg}}$ is limited by its bidding quantity $\overline{Q_t^{\text{agg}}}$ in Constraint (18). Note that the aggregator is the only market participant offering demand flexibility.

3) Demand Response Provider's Perspective: Each residential DR provider h (set $\mathcal{H}$) minimizes the cost of electric space heating and hot water production while maintaining thermal comfort, leveraging the inherent thermal inertia of the building shell and the hot water tank to shift its electricity consumption $d_{h,t}^{\text{H}}$ to periods of low electricity prices $\lambda_{h,t}^{\text{A}}$:

$$\text{Min.} \sum_{t\in\mathcal{T}} \lambda_{h,t}^{\text{A}} \cdot d_{h,t}^{\text{H}} \tag{19}$$

subject to

$$\boldsymbol{\theta_{h,t}} - \boldsymbol{\theta_{h,t-1}} = \boldsymbol{\mathcal{G}}(d_{h,t}^{\text{H}}, C_h, \overline{P_h}, A_h, E_{h,t}),\ \forall t \in \mathcal{T} \tag{20}$$

$$\underline{\boldsymbol{\theta_{h,t}}} \leq \boldsymbol{\theta_{h,t}} \leq \overline{\boldsymbol{\theta_{h,t}}},\ \forall t \in \mathcal{T} \tag{21}$$

Each building is equipped with a heat pump and an auxiliary heater. For sake of brevity, and to allow the reader to focus on the interaction between the agents, we summarize the linear model relating the electricity demand $d_{h,t}^{\text{H}}$ to the change in hot water and indoor air temperatures $\boldsymbol{\theta_{h,t}}$ as $\mathcal{G}$ in Eq. (20). This set of linear equations depends on, i.a., the state-space model A_h, the nameplate capacity $\overline{P_h}$ of the electric heating systems, their coefficient of performance C_h and external

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disturbances $E_{h,t}$, such as the ambient temperature. Constraint (21) enforces the user-defined comfort constraints ($\underline{\boldsymbol{\theta_{h,t}}}$, $\overline{\boldsymbol{\theta_{h,t}}}$) imposed on, i.a., the indoor air temperature. A full description of the demand response provider model is available in [3], [29], [36].

4) Aggregator - Market Interaction: The aggregator acts as a price-maker in the market by bidding strategically, leveraging the flexibility of its consumers to minimize its electricity procurement cost. Given this relation and neglecting the interaction with the DR providers, the problem can be represented as a bilevel optimization problem:

$$\text{Max.} \quad (8)$$

subject to

$$\text{Aggregator constraints:} \quad (11) - (12)$$

$$\text{Market clearing:} \quad \text{Max.} \quad (13) \quad \text{s.t.} \quad (14) - (18)$$

The MPEC equivalent of the bilevel model above is obtained by extending the lower-level market clearing problem (Eq. (13) - (18)) with its KKT conditions (Eq. (22)-(33), $\forall \omega \in \Omega$):

Stationarity conditions:

$$P^{\mathrm{d}} - \lambda_{t,\omega} - \overline{\epsilon_{t,\omega}} + \underline{\epsilon_{t,\omega}} = 0, \quad \forall t \in \mathcal{T} \tag{22}$$

$$P^{\mathrm{agg}} - \lambda_{t,\omega} - \overline{\kappa_{t,\omega}} + \underline{\kappa_{t,\omega}} = 0, \quad \forall t \in \mathcal{T} \tag{23}$$

$$- P_i^{\mathrm{g}} + \lambda_{t,\omega} - \overline{\delta_{i,t,\omega}} + \underline{\delta_{i,t,\omega}} = 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{24}$$

$$\lambda_{t,\omega} - \overline{\zeta_{t,\omega}} + \underline{\zeta_{t,\omega}} = 0, \quad \forall t \in \mathcal{T} \tag{25}$$

Complementary slackness conditions:

$$0 \leq g_{i,t,\omega} \perp \underline{\delta_{i,t,\omega}} \geq 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{26}$$

$$0 \leq (\overline{G_i} - g_{i,t,\omega}) \perp \overline{\delta_{i,t,\omega}} \geq 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{27}$$

$$0 \leq d_{t,\omega} \perp \underline{\epsilon_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{28}$$

$$0 \leq (\overline{D_t} - d_{t,\omega}) \perp \overline{\epsilon_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{29}$$

$$0 \leq w_{t,\omega} \perp \underline{\zeta_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{30}$$

$$0 \leq (\overline{W_t} - w_{t,\omega}) \perp \overline{\zeta_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{31}$$

$$0 \leq q^{\mathrm{agg}}_{t,\omega} \perp \underline{\kappa_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{32}$$

$$0 \leq (\overline{Q^{\mathrm{agg}}_t} - q^{\mathrm{agg}}_{t,\omega}) \perp \overline{\kappa_{t,\omega}} \geq 0, \quad \forall t \in \mathcal{T} \tag{33}$$

5) Aggregator – DR Provider Interaction: We distinguish between three interaction mechanisms:

a) Retailer: The aggregator offers a fixed rate λ^{A} to the DR provider, effectively acting as a conventional retailer. This leads to a complete decoupling between the retailer's procurement problem, as described in Section II-C4, and the consumers problem (Eqs. (19)–(21) with $\lambda^{\mathrm{A}}_{h,t} = \lambda^{\mathrm{A}}$). Consumers minimize their energy demand $d^H_{h,t}$, which is considered a parameter in the retailer's problem. As a reference case, we assume that the retailer does not make any profit, i.e., the consumers are charged a retail rate that allows the retailer to recover its costs, resulting in energy cost $R^{\mathrm{R}}_h = \sum_{t\in\mathcal{T}} \lambda^A \cdot d^{\mathrm{H}}_{h,t}$ for consumer h:

$$\sum_{h\in\mathcal{H}} NB_h \cdot R^{\mathrm{R}}_h = \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega} \tag{34}$$

b) Stackelberg Game: Alternatively, one can describe the interaction between the DR provider and the aggregator as a Stackelberg Game, governed by the time-varying and consumer-specific retail rate $\lambda^{\mathrm{A}}_{h,t}$ offered by the aggregator:

$$\text{Maximize} \sum_{t\in\mathcal{T}} \sum_{h\in\mathcal{H}} NB_h \cdot \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} - \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega} \tag{35}$$

subject to

$$\sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} \leq R^{\mathrm{R}}_h - \underline{\Delta R^{\mathrm{R}}_h}, \quad \forall h \in H \tag{36}$$

$$\text{Aggregator constraints: } (11) - (12)$$

$$\text{Market clearing: } (14) - (18), \ (22) - (33)$$

$$\text{Demand response problem: Min.} \quad (19) \quad \text{s.t.} \quad (20) - (21)$$

Constraint (36) ensures that the cost for the consumer is at least $\underline{\Delta R^{\mathrm{R}}_h}$ lower than that under the profit-neutral retailer paradigm. $\underline{\Delta R^{\mathrm{R}}_h}$ may be interpreted as the required monetary benefit, i.e., the expected savings, for a consumer h to sign a contract with an aggregator. In this paper, we obtain R^{R}_h by first solving the retailer problem (Section II-C5a) to inform Constraint (36). In practice, consumers may provide (an estimate of) this number based on, e.g., the previous billing period.

Note furthermore that we do not make any assumptions w.r.t. the ownership of the infrastructure that unlocks the flexibility of the DR loads (e.g., the smart thermostat). Higher degrees of consumer-ownership may however lead to higher lower limits on the required cost savings $\underline{\Delta R^{\mathrm{R}}_h}$.

The resulting bilevel problem can be recast as an MPEC by replacing each of the lower-level DR providers' problems (19)–(21) with their KKT conditions, but solving the resulting MPEC may come at a significant computational cost. However, as we will show below, if a solution of this Stackelberg Game exists, it belongs to the set of solutions of the Nash Bargaining Game under specific assumptions w.r.t. the bargaining power of the consumers and the aggregator. Determining the conditions under which these Stackelberg equilibria exist is however out of the scope of this paper.

c) Nash Bargaining: In this paradigm, DR providers engage in a long-term cooperation with the aggregator. The monetary benefits of this cooperation are split according to the outcome of a Nash Bargaining Game, which explicitly captures the power relations between the aggregator and the consumers. Although this cooperative game is played well ahead of day-to-day operations described in this paper, this paradigm allows representing the DR providers and the aggregator as a single entity participating in the electricity market, following [21]. To formalize this cooperative game, we define the cooperation and disagreement strategies for the aggregator and the DR providers [19], [20]:

- *Cooperation strategies:* A DR provider will sign a contract with an aggregator if this reduces its current electricity bill R^{R}_h by at least $\underline{\Delta R^{\mathrm{R}}_h}$, i.e., if $\sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} \leq R^{\mathrm{R}}_h - \underline{\Delta R^{\mathrm{R}}_h}$. In this case, each consumer plays its cooperation strategy $S^{\mathrm{H}}_h = (d^{\mathrm{H}}_h)$, defined by the set of inequalities in Eq. (20)-(21). The DR provider's benefit $\mathcal{B}^{\mathrm{H}}_h$ is defined as the change in the consumer's electricity bill, i.e.,

$$\mathcal{B}^{\mathrm{H}}_h = R^{\mathrm{R}}_h - \sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t}, \ \forall h \in \mathcal{H} \tag{37}$$

As before, we obtain R^{R}_h by solving the retailer problem (Section II-C5a). Similarly, an aggregator will play its cooperation strategy $S^{\mathrm{A}} = (\lambda^{\mathrm{A}}_{h,t})$ if this results in a profit that exceeds a certain lower bound, i.e., if $\sum_{h\in\mathcal{H}} NB_h \cdot (\sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} - \underline{\Delta R^{\mathrm{A}}_h}) \geq \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega}$. The aggregator's

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benefit $\mathcal{B}^{\mathrm{A}}$ resulting from this cooperation is defined as

$$\mathcal{B}^{\mathrm{A}} = \sum_{h\in\mathcal{H}} NB_h \cdot \sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} - \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega} \quad (38)$$

- *Disagreement strategies:* If the offered retail rate $\lambda^{\mathrm{A}}_{h,t}$ would not decrease the DR provider's electricity bill R^{R}_h by at least $\underline{\Delta R^{\mathrm{R}}_h}$, a consumer will not sign a contract with the aggregator. Hence, the DR provider plays its disagreement strategy $S^{\mathrm{H}*}_h = (0)$, which results in net benefit $\mathcal{B}^{\mathrm{H}*}_h = 0$. Similarly, if the retail rate required by the consumer to engage in a cooperation would not result in a profit of at least $\underline{\Delta R^{\mathrm{A}}_h}$ for the aggregator, the aggregator will not engage in a contract with said consumer (disagreement strategy $S^{\mathrm{A}*} = (0)$ and $\mathcal{B}^{\mathrm{A}*} = 0$).
- *Benefit split based on Nash Bargaining:* If a bargaining solution $(d^H_{h,t}, \lambda^A_{h,t}) \in \langle [S^{\mathrm{H}}_h, S^{\mathrm{A}}], [S^{\mathrm{H}*}_h, S^{\mathrm{A}*}] \rangle$ exists in which the previous conditions hold, the benefit of this cooperation is split according to the solution of the following optimization problem [19], [20]:

$$\text{Max.} \prod_{h\in\mathcal{H}} \left[\mathcal{B}^H_h - \mathcal{B}^{H*}_h\right]^{y^{\mathrm{H}}_h} \cdot \left[\mathcal{B}^A - \mathcal{B}^{A*}\right]^{y^{\mathrm{A}}} \quad (39)$$

with y^{H}_h and y^{A} the exogenous bargaining power of consumer h and the aggregator ($y^{\mathrm{H}}_h, y^{\mathrm{A}} \in [0,1]$, $\sum_{h\in\mathcal{H}} y^{\mathrm{H}}_h + y^{\mathrm{A}} = 1$). The bargaining power of a consumer and the aggregator may depend on, e.g., the information each entity has available and the ownership structure of the DR infrastructure. Using the Pareto-efficiency axiom, this objective may be recast as the division of the maximum attainable benefit $\mathcal{B}$ [21]. Assuming the aggregator engages in bilateral negotiations with each DR provider and all actors are risk-neutral [21], the overall monetary benefits $\mathcal{B}$ of the aggregator - DR provider cooperation are given by

$$\begin{aligned}\mathcal{B} &= \mathcal{B}^A + \sum_{h\in\mathcal{H}} NB_h \cdot \mathcal{B}^H_h \quad (40)\\ &= \sum_{h\in\mathcal{H}} NB_h \cdot R^{\mathrm{R}}_h - \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega}\end{aligned}$$

With division factors x^{H}_h and x^{A} ($\sum_{h\in\mathcal{H}} x^{\mathrm{H}}_h + x^{\mathrm{A}} = 1$, $x^{\mathrm{H}}_h,\ x^{\mathrm{A}} \in [0,1]$, $\mathcal{B}^{\mathrm{H}}_h := x^{\mathrm{H}}_h \cdot \mathcal{B},\ \forall h \in \mathcal{H}$, $\mathcal{B}^{\mathrm{A}} := x^{\mathrm{A}} \cdot \mathcal{B}$) and the zero benefit associated with the disagreement strategies ($\mathcal{B}^{H*}_h = 0,\ \forall h \in \mathcal{H}$, $\mathcal{B}^{A*} = 0$), the Bargaining Game (Eq. (39)) can be formally described by the following optimization problem [19]–[21]:

$$\text{Max.} \prod_{h\in\mathcal{H}} \left[x^{\mathrm{H}}_h \cdot \mathcal{B}\right]^{y^{\mathrm{H}}_h} \cdot \left[x^{\mathrm{A}} \cdot \mathcal{B}\right]^{y^{\mathrm{A}}} \quad (41)$$

In this paper, we assume that the aggregator treats all DR providers nondiscriminatory ($x^{\mathrm{H}}_h = x^{\mathrm{H}}$) and that all consumers (N in total) have equal bargaining power ($y^{\mathrm{H}}_h = y^{\mathrm{H}}$). Objective (41) can be recast as:

$$\text{Max.} \quad (x^{\mathrm{H}})^{y^{\mathrm{H}} \cdot N} \cdot (1 - x^{\mathrm{H}} \cdot N)^{1 - y^{\mathrm{H}} \cdot N} \cdot \mathcal{B} \quad (42)$$

Furthermore, the bargaining process on the formation of retail rates, i.e., the division factors x^{H}_h and x^{A}, governing the split of the overall benefit of DR, takes place well before the aggregator offers demand in the DAM[2]. For a given benefit $\mathcal{B}$ and assuming N consumers with the same bargaining power y^{H}, it is trivial to show that the Nash Bargaining Game (42) allows three solutions:

1) $x^{\mathrm{H}} = 0$, i.e., all benefits go to the aggregator;
2) $x^{\mathrm{H}} = \frac{1}{N}$, i.e., all benefits go to the consumers and are distributed proportionally;
3) $x^{\mathrm{H}} = y^{\mathrm{H}} = \frac{1-y^{\mathrm{A}}}{N}$, i.e., the benefit is split between the aggregator and consumers according to their bargaining powers.

Note that solution 3) coincides with solution 2) if the aggregator has no bargaining power ($y^{\mathrm{A}} = 0$), whereas solution 3) is identical to solution 1) when $y^{\mathrm{A}} = 1$. Solution 1) reflects the outcome of the Stackelberg Game between consumers and the aggregator (see below).

Hence, assuming a solution to this Bargaining Game exists, the only term the aggregator can optimize in objective (42) on a *daily basis* is $\mathcal{B}$ (Eq. (40)), which yields the following optimization problem governing the aggregator's day-to-day operations:

$$\text{Max.} \quad \sum_{h\in\mathcal{H}} NB_h \cdot R^{\mathrm{R}}_h - \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} \pi_\omega \cdot \lambda_{t,\omega} \cdot q^{\mathrm{agg}}_{t,\omega} \quad (43)$$

subject to

Aggregator constraints: (11) – (12)

Demand response constraints: (20) – (21)

Market clearing: (14) – (18), (22) – (33)

The inclusion of constraints (20)-(21) must be interpreted as a description of the pre-agreed set of DR providers' strategies $d^{\mathrm{H}}_{h,t}$. Similarly, explicit constraints on the price charged to the DR providers (the aggregator's strategy) may be enforced.

Note that, in the specific, hypothetical case that consumers do not have bargaining power ($y^{\mathrm{H}}_h = 0$, $y^{\mathrm{A}} = 1$), the aggregator may claim all the benefits of the cooperation with the DR providers up to the point that the conditions of the consumers to play their agreement strategies are strictly binding ($x^{\mathrm{A}} = 1 - \sum_{h\in\mathcal{H}} \frac{NB_h \cdot \underline{\Delta R^{\mathrm{R}}_h}}{\mathcal{B}}$). Without explicit constraints on the formation of the retail rate $\lambda^{\mathrm{A}}_{h,t}$, the aggregator is always able to define a retail rate such that its revenue is maximized and, consequently, Constraint (36) is binding. Under these assumptions, (i) Objectives (43) and (35) are identical and (ii) the objectives of the demand providers problems (Eq. (19)) are fixed. Hence, the optimization problems describing the Stackelberg Game and the Nash Bargaining Game are effectively solved on the same feasibility sets, given by Eq. (11)–(12), (20)–(21), (14)–(18) and (22)–(33). Consequently, if a solution of the Stackelberg Game between the aggregator and its consumers exists, it is included in the set of possible outcomes of the Nash Bargaining Game[3].

In what follows, we will therefore focus on (i) the overall benefit $\mathcal{B}$ of the DR provider-aggregator cooperation under the Nash Bargaining paradigm and (ii) the maximally attainable benefit per consumer, i.e., solution 2) of the Nash Bargaining Game above.

[2]The bargaining game may be played based on an estimate of $\mathcal{B}$, since the benefit of the aggregator–DR provider cooperation is only revealed in real time. The outcome of the bargaining game entails some restrictions on the retail tariff (i.e., such that $\sum_{t\in\mathcal{T}} \lambda^{\mathrm{A}}_{h,t} \cdot d^{\mathrm{H}}_{h,t} = x^H_h \cdot \mathcal{B}$), but does not prescribe its *time-varying magnitude*. In addition to the distribution of the benefits of DR, the tariff may be used, e.g., as a control signal for the DR loads.

[3]This statement should not be generalized. For example, if the consumers own battery energy storage systems, the Stackelberg and Nash Bargaining Game may lead to different solutions [21]. In this specific case, this statement holds as consumers must use their heating system to maintain thermal comfort.

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6) Equivalent MIQCP formulations:

a) Retailer: The retailer minimizes its expenses in the DAM and determines a flat retail rate λ^{A} to ensure profit-neutrality (34):

$$\text{Max.} \quad -\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega}\pi_{\omega}\cdot\lambda_{t,\omega}\cdot q^{\mathrm{agg}}_{t,\omega} \tag{44}$$

subject to

$$\text{Aggregator constraints: (11)} - \text{(12)},\ \text{(34)}$$
$$\text{Market clearing: (14)} - \text{(18)},\ \text{(22)} - \text{(33)}$$

Recall that $d^{\mathrm{H}}_{h,t}$ is a parameter in the retailer's problem, obtained from solving the DR provider problems (Eqs. (19)-(21)) separately, assuming a flat retail rate λ^{A}.

b) Stackelberg Game/Nash Bargaining Game: Neglecting the first, fixed term in objective (43), the aggregator's problem can described as

$$\text{Max.} \quad -\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega}\pi_{\omega}\cdot\lambda_{t,\omega}\cdot q^{\mathrm{agg}}_{t,\omega} \tag{45}$$

subject to

$$\text{Aggregator constraints: (11)} - \text{(12)}$$
$$\text{Demand response constraints: (20)} - \text{(21)}$$
$$\text{Market clearing: (14)} - \text{(18)},\ \text{(22)} - \text{(33)}$$

Each MPEC above contains non-linearities in (i) the objective of the aggregator/retailer and (ii) the complementary slackness conditions (Eqs. (26)–(33)). Via stationarity condition (23) and complementary slackness conditions (32)–(33), we recast the non-linear term $\lambda_{t,\omega}\cdot q^{\mathrm{agg}}_{t,\omega}$ in the objective as:

$$\sum_{t\in\mathcal{T}}\lambda_{t,\omega}\cdot q^{\mathrm{agg}}_{t,\omega}=\sum_{t\in\mathcal{T}}\left[P^{\mathrm{agg}}\cdot q^{\mathrm{agg}}_{t,\omega}-\overline{Q^{\mathrm{agg}}_{t}}\cdot\overline{\kappa_{t,\omega}}\right], \quad \forall\omega\in\Omega \tag{46}$$

The last term in the equation above is non-linear, but can be expressed as a sum of linear terms by applying the strong duality theorem on the market clearing problem [37]. This theorem states that, under certain conditions which are satisfied for linear optimization problems such as market clearing problem (13)-(18), optimal solutions to the primal and the associated dual problem yield the same objective value [37]. This equality is expressed in Eq. (47), which, after isolating $\sum_{t\in\mathcal{T}}\overline{\kappa_{t,\omega}}\cdot\overline{Q^{\mathrm{agg}}_{t}}$, reads:

$$\sum_{t\in\mathcal{T}}\overline{\kappa_{t,\omega}}\cdot\overline{Q^{\mathrm{agg}}_{t}}=\sum_{t\in\mathcal{T}}\Big(P^{\mathrm{d}}\cdot d_{t,\omega}+P^{\mathrm{agg}}\cdot q^{\mathrm{agg}}_{t,\omega}-\sum_{i\in\mathcal{I}}P^{\mathrm{g}}_{i}\cdot g_{i,t,\omega} - \sum_{i\in\mathcal{I}}\overline{G_i}\cdot\overline{\delta_{i,t,\omega}}-\overline{D_t}\cdot\overline{\epsilon_{t,\omega}}-\overline{W_{t,\omega}}\cdot\overline{\zeta_{t,\omega}}\Big), \quad \forall\omega\in\Omega \tag{47}$$

The non-linearities in the complementary slackness conditions (26)–(33) can be linearized using the method of Fortuny-Amat and McCarl [38]. For example, Eq. (26) can be linearized as follows, with M an arbitrary, sufficiently large constant:

$$0\leq\underline{\delta_{i,t,\omega}}\leq M\cdot\underline{\delta^{*}_{i,t,\omega}}, \quad \forall i\in\mathcal{I},t\in\mathcal{T},\omega\in\Omega \tag{48}$$

$$0\leq g_{i,t,\omega}\leq M\cdot(1-\underline{\delta^{*}_{i,t,\omega}}), \quad \forall i\in\mathcal{I},t\in\mathcal{T},\omega\in\Omega \tag{49}$$

$$\underline{\delta^{*}_{i,t,\omega}}\in\{0,1\}, \quad \forall i\in\mathcal{I},t\in\mathcal{T},\omega\in\Omega \tag{50}$$

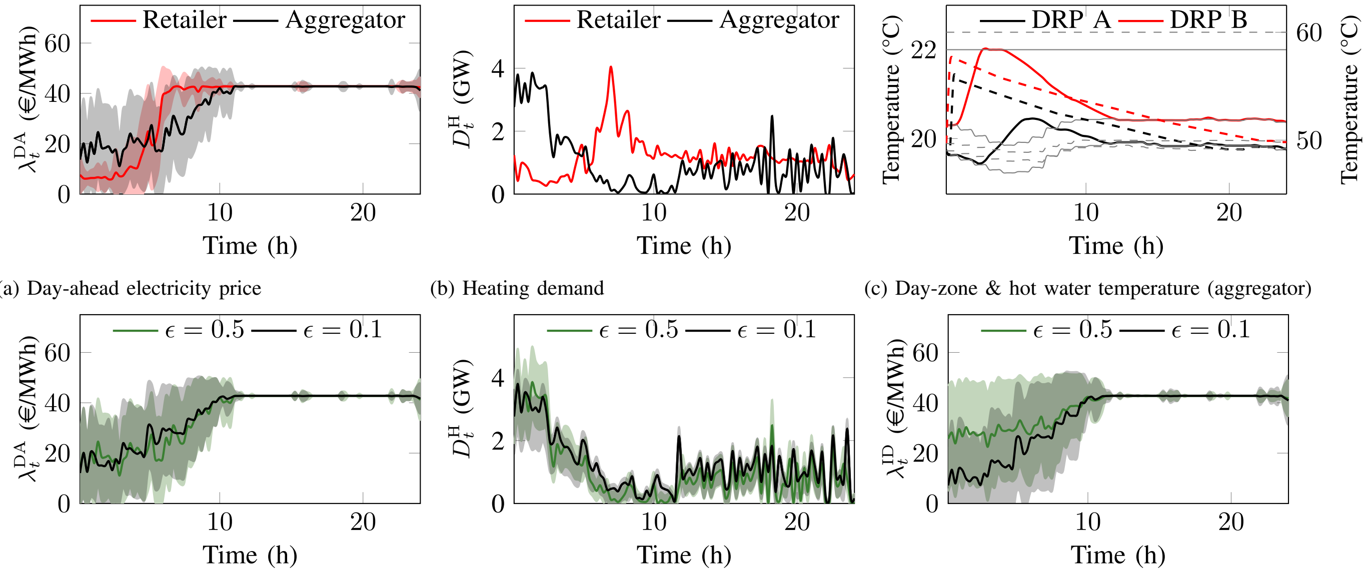
Leveraging the reformulation of the chance constraint and these linearizations, one obtains two MIQCP problems that can be solved with commercial solvers (Cplex, Gurobi) [34].

III. Case Study

We study DR with residential electric heating systems using a model inspired by the current Belgian power system. Wind energy, the only source of uncertainty in the DAM, is assumed to cover 50% of the annual energy demand prior to the introduction of electric space heating. Seven GW of gas-fired (six GW CCGTs, one GW OCGTs) and one GW of oil-fired generation are added to the system to cover the additional heating demand. Grid constraints and interconnections with neighboring countries are neglected. The number of buildings is set to $\sum_{h\in\mathcal{H}}NB_h = 10^6$ and the building portfolio is represented by an average 2030 low-energy building described in [26]. The temperature bounds $(\overline{\theta_{h,t}},\ \underline{\theta_{h,t}})$ aim to represent possible occupant behavior (see [26] and references therein). Conventional generation is assumed to offer its entire capacity at average generation cost. The aggregator and conventional demand bid at 1000 €/MWh (i.e., the price cap). To ensure consistency, all time series (e.g., demand, weather data) are based on data for the year 2013.

Wind power scenarios, describing the uncertainty of available wind power in the DAM, are generated using a data-driven scenario generation technique [36]. Using a modified fast forward scenario reduction inspired by [36], sets of 30 scenarios are selected from a large set, which yield in- and out-of-sample stable solutions. To calculate an expected procurement cost in the DAM, out-of-sample evaluations are executed as Monte Carlo market clearing simulations for a new, large set of possible wind power scenarios, taking the bids of the aggregator and the available wind power as given. We execute an additional set of Monte Carlo market clearing simulations for each DAM outcome to test the ability of an aggregator to adapt its day-ahead position in intra-day, real-time and balancing markets to mitigate DR controllability issues. The day-ahead position of the aggregator and the day-ahead expected heating demand schedule $\sum_{h\in\mathcal{H}}NB_h\cdot d^{\mathrm{H}}_{h,t}$, corrected for a disturbance sampled from the distributions considered in the chance constraints to mimic the impact of limited controllability[4], are assumed to be fixed. The aggregator acts as a price-taker in the IDM. If the aggregator procured too much (little) energy on the DAM, the aggregator must sell (buy) the excess (deficit) on the IDM. Only RES-based generation and flexible gas/oil-fired generation assets are capable adapting their output intra-day. To mimic possible divergence between intraday and day-ahead prices, we perform a sensitivity analysis in which the aggregator pays 110% of the intraday price for deficits and sells a possible surplus at 90% of the intraday price. In all cases, sufficient capacity is available in the IDM to ensure that the aggregator can meet the real-time heating load, in order to avoid thermal discomfort for the DR provider. As such, these simulations yield an estimate of the operating costs an aggregator may save (incur) by selling (buying) excess (deficits) in intra-day, real-time and balancing markets, which allows estimating the optimal risk behavior (ϵ) of the aggregator and its robustness to our IDM assumptions. The DAM and IDM results are obtained from large sets of market clearing problems, formulated as in Eq. (13)–(18).

[4] We will refer to these demand profiles as the real-time heating load.



(a) Day-ahead electricity price (b) Heating demand (c) Day-zone & hot water temperature (aggregator)

(d) Day-ahead electricity price (aggregator) (e) Heating demand (aggregator) (f) Intra-day electricity price (aggregator, no premium)

Fig. 2. The average (solid lines) and range of DAM clearing prices (Fig. 2a and 2d), heating demand (Fig. 2b and 2e), the expected day-zone (solid lines) and hot water (dashed lines) temperatures for two DR providers ('DRP') under the aggregator paradigm (Fig. 2c) and the average and range of IDM clearing prices (no mark-ups considered) (Fig. 2f) on day 316. The lower and upper bound of the range in Fig. 2a, 2d and 2f indicate the the expected market clearing price plus or minus one standard deviation of the observed market clearing prices at each time step. The grey lines in Fig. 2c indicate the lower and upper comfort constraints of the two DR providers. In the top row, the DR loads are assumed to be perfectly controllable. In the bottom row, the DR loads control error is assumed to be characterized by $\delta^{\mathrm{NP}} = N(0, 100\text{ MW})$ and $\delta^{\mathrm{P}} = N(0, 0.1)$. The resulting heating demand bid (solid lines, Fig. 2e) and range of possible heating demand realizations (shaded areas, Fig. 2e) is dependent on the risk attitude of the aggregator (ϵ). By definition, setting ϵ equal to 0.5 yields the same day-ahead heating bid and market outcome as under the assumption of perfect controllability.

In Section III-A, we show how an aggregator participates in the DAM and IDM on day 316 of the calendar year with perfectly and imperfectly controllable DR loads. Subsequently, we provide a sensitivity analysis of these metrics w.r.t. our assumptions on σ^{P}, σ^{NP} and ϵ, based on simulations for seven days throughout the heating season, selected based on the daily average ambient temperature. The 16th, 40th, 48th, 316th, 262th, 105th and 295th day of the calendar year, corresponding to the 1st, 10th, 25th, 50th (median), 75th, 90th and 99th percentile of the distribution of the average daily ambient temperature, were selected[5].

A. Managing market uncertainty & limited DR controllability

As a reference case, we study the retailer's and aggregator's bidding behavior assuming perfectly controllable DR resources (Fig. 2). On this particular day, low demand periods during the night coincide with abundant wind power output, resulting in low market clearing prices (Fig. 2a). Under the retailer paradigm, consumers minimize their own energy use, which results in a strong peak in the heating demand in the morning (Fig. 2b). This in turn triggers possible price peaks in the DAM in hours 7-10 (light red area in Fig. 2a). Despite the significantly lower heating demand during the rest of the day and considerable wind power uncertainty, prices remain stable at around 42 €/MWh (i.e., the assumed average generation cost of gas-fired power plants).

If the aggregator is allowed to shift the heating demand within the user-specified comfort constraints, the morning price peaks are mitigated (Fig. 2a). This is the result of shifting the heating demand to earlier hours (Fig. 2b), which results in significant preheating of the considered dwellings (Fig. 2c). This is evident from the difference between, e.g., the expected day-zone temperature (solid) and hot water temperatures (dashed) and the associated comfort constraints (grey lines, Fig. 2c). The shift in demand leads to higher average prices during the night (Fig. 2a). The aggregator succeeds in limiting the expected price well below 42 €/MWh, whereas the retailer must tolerate an expected price of approximately 42 €/MWh during the morning peak. Overall, this leads to a monetary benefit $\mathcal{B}$ (Eq. (40)) between 0.33 M€ and 2.49 M€, depending on the DAM outcome, with an expected value of 1.66 M€ (which is to be shared by the aggregator and the DR providers). If consumers have all bargaining power, the expected saving per consumer equals 1.66€ on this particular day.

If the DR resource is limitedly controllable, the DR aggregator may procure more than the expected heating demand to deal with unexpected real-time deviations from the expected heating demand and to avoid buying significant volumes of energy in the IDM at a premium. Based on the distributions describing the DR resources' limited controllability and its risk parameter ϵ, the aggregator is able to define a heating demand bid (solid lines, Fig. 2e) and a range of possible realizations of the heating demand at each time step (shaded areas, Fig. 2e). By tolerating a higher heating demand during most of the day, the risk-averse aggregator ($\epsilon = 0.1$) is able to approximately maintain its heating bid and expected market prices during the night (Fig. 2d). Note that, by definition, the risk-neutral aggregator and the aggregator not considering limited controllability provide the same heating bids (Fig. 2b

[5]Other methods exist to select 'representative' days, e.g., [39], [40].

and 2e) and anticipate the same market clearing outcomes (Fig. 2a and 2d). Compared to a retailer with a perfectly controllable and predictable heating load, the overall benefit $\mathcal{B}$ (Eq. (40)) in the DAM is limited to 0.70 M€ or at most 0.7 € per consumer if the aggregator is risk-averse. Note that the risk-averse aggregator may incur a loss under some DAM outcomes: the benefit $\mathcal{B}$ ranges between -0.70 M€ and 1.72 M€.

After closure of the DAM, the aggregator may sell excess and buy shortages of electrical energy on intra-day and balancing markets to match the heating demand of its consumers. The risk-neutral aggregator faces higher intra-day prices during off-peak periods (green line, Fig. 2f) and must procure larger volumes compared to the risk-averse aggregator. The risk-averse aggregator may sell excess energy, but is more likely to receive a lower price for this energy in the IDM (black line, Fig. 2f). In absence of premiums on intra-day prices, the expected net procurement cost in the IDM equals 0.08 M€ for the risk-neutral aggregator, which leads to an overall expected benefit $\mathcal{B}$ of 1.58 M€ compared to the risk-neutral retailer with a perfectly controllable heating load. The risk-averse aggregator secures an expected intraday profit of 0.72 M€ and a combined expected benefit of 1.42 M€. Note, however, that these intra-day profits may exhibit a significant spread: e.g., the risk-averse aggregator's intra-day profit ranges from 0.49 M€ to 1.04 M€, depending on the real-time heating load. With a 10% mark-up (if the aggregator buys energy in the IDM) or mark-down (if the aggregator sells energy in the IDM) on IDM prices, the expected procurement cost in the IDM increases to 0.12 M€ for the risk-neutral aggregator, whereas the risk-averse aggregator secures a lower profit of 0.65 M€. Indeed, premiums on IDM prices lead to (i) higher expenses to cover deficits and (ii) lower revenues if the aggregator has procured too much energy in the DAM. Overall, however, the aggregator still secures an operating profit of 1.34 M€ (risk-averse aggregator) to 1.51 M€ (risk-neutral aggregator). In very extreme conditions (e.g., low wind power availability and high heating demand), insufficient capacity may be available intra-day to meet the load. In this case study, this does not occur. Consequently, the consumer attains savings of at most 1.58 € on this particular day (risk-neutral aggregator, no IDM price mark-ups).

This example shows that a DR aggregator may secure operating profits by performing price arbitrage in uncertain DAM, even if the DR resource exhibits some limited controllablity. In the next section, we examine whether some optimal trade-off exists between the risk parameter ϵ, and thus the procured volume in the DAM, and the volume procured in the IDM as a function of the degree of limited controllability of the DR resource (σ^{P}, σ^{NP}).

B. Aggregator profitability

Table I summarizes the results of extensive numerical simulations for seven days selected across the heating season based on the average outside temperature, for three sets of σ-values (σ^{NP} and σ^{P}) describing the degree of limited controllability of the DR providers and six ϵ-values as a measure of the risk attitude of the aggregator. Larger σ-values indicate DR loads with poorer controllability. Smaller ϵ-values represent more risk-averse aggregator behavior. The values indicated with an asterisk ($\mathcal{B}^*$, ID*) are obtained considering a 10% mark-up (if the aggregator sells energy) or mark-down (if the aggregator buys energy) on IDM prices. Several trends can be identified.

First, the overall benefit of DR remains, in the majority of cases, approximately unchanged. Only if the DR providers become less controllable and/or the aggregator becomes strongly risk-averse ($\epsilon \rightarrow 0$), the benefits decrease. A risk-averse aggregator procures more electricity in the DAM, driving down its benefit w.r.t. the retailer in the DAM ('DA' in Table I). In most heating demand realizations, part of this procured volume must be sold in the IDM. These IDM revenues compensate for the higher expenses of risk-averse aggregators in the DAM, especially if prices between IDM and DAM are well correlated. However, if we impose premiums on IDM prices, which may be the case in less liquid IDMs, we observe, as expected, an overall reduction in the attainable benefit from $\mathcal{B}$ to $\mathcal{B}^*$. In this case, aggregators face lower (while selling) or higher (while buying energy) prices in the IDM. Risk-averse aggregators, who have to compensate for their higher expenses in the DAM by (predominantly) selling surplus energy in the IDM, see lower revenues in the IDM. This, in turn, reduces the overall attainable benefit $\mathcal{B}^*$. However, the expected volumes traded in the IDM by the aggregator are typically an order of magnitude smaller than those traded in the DAM. Only if the heating load is low (e.g., on day 105 and day 295), the non-proportional limited controllability term δ^{NP} dominates the aggregator's bidding strategy and the IDM volumes are similar to the DAM volumes. On these days, the aggregator's bid represents a very low share of the total DAM volume. High volumes sold in the IDM may drive down IDM clearing prices, which may result in insufficient intra-day revenues to fully compensate the day-ahead loss ('ID' in Table I). Nevertheless, in most cases, the aggregator succeeds in securing a profit ($\mathcal{B} \geq 0$, $\mathcal{B}^* \geq 0$).

Second, as the aggregator becomes more risk-averse, the share of intra-day revenues in the overall benefits strongly increases. Securing these profits requires liquid and well-functioning intra-day and real-time markets (see above). Assuming a less liquid IDM, these revenues ID*, which are highest on days with a high heating demand and for risk-averse aggregators, decrease significantly, thereby reducing the attainable benefit $\mathcal{B}^*$. Note that we assume that the amount of generation capacity available in the IDM is significantly larger than the deficits/excesses the aggregator must compensate in all cases. If insufficient generation capacity would be available in the IDM, this may lead to thermal discomfort for consumers, which in turn may need to be financially compensated.

Third, as the heating demand decreases, as indicated by the increasing percentiles in Table I, the overall benefit $\mathcal{B}$ typically decreases. In these cases, the non-proportional limited controllability term δ^{NP} gains relative importance (see above) and the overall benefit may turn negative for risk-averse aggregators. As expected, the aforementioned effect is more pronounced if the IDM is less liquid, which offers the aggregator less opportunities to balance its portfolio.

TABLE I. Change in the benefit of the DR provider-aggregator cooperation $\mathcal{B}$ for different ϵ and σ^{P}, σ^{NP}-values, split between the day-ahead (DA) benefit (as defined in Eq. (40)) and the intra-day (ID) correction. The values indicated with an asterisk ($\mathcal{B}^*$, ID^*) are obtained considering a 10% mark-up (if the aggregator sells energy) or mark-down (if the aggregator buys energy) on IDM prices. The corresponding percentile in the daily average of the ambient temperature – as an approximation of the heating demand on that day – is indicated between parentheses. Day 16 is the coldest considered day, day 295 the hottest. The green cells indicate the maximum benefit $\mathcal{B}$. Red cells indicate negative operating profits ($\mathcal{B} \leq 0$).

Day		$\sigma^{\mathrm{P}} = 0.05$ & $\sigma^{\mathrm{NP}} = 50$ MW						$\sigma^{\mathrm{P}} = 0.1$ & $\sigma^{\mathrm{NP}} = 100$ MW						$\sigma^{\mathrm{P}} = 0.15$ & $\sigma^{\mathrm{NP}} = 150$ MW					
↓	$\epsilon \rightarrow$	0.5	0.4	0.3	0.2	0.1	0.01	0.5	0.4	0.3	0.2	0.1	0.01	0.5	0.4	0.3	0.2	0.1	0.01
16 (1%)	$\mathcal{B}$ (M€)	2.63	2.63	2.62	2.60	2.61	2.54	2.63	2.64	2.56	2.53	2.54	2.32	2.61	2.58	2.57	2.52	2.40	1.84
	$\mathcal{B}^*$ (M€)	2.59	2.57	2.55	2.52	2.51	2.36	2.56	2.54	2.44	2.38	2.33	1.96	2.49	2.43	2.39	2.30	2.09	1.29
	DA (M€)	2.64	2.43	2.21	1.94	1.61	0.73	2.64	2.25	1.75	1.22	0.54	-1.31	2.64	2.01	1.36	0.57	-0.60	-3.64
	ID (M€)	-0.002	0.20	0.41	0.66	1.00	1.81	-0.01	0.39	0.81	1.31	2.00	3.63	-0.03	0.57	1.21	1.95	3.00	5.48
	ID* (M€)	-0.05	0.14	0.35	0.58	0.90	1.63	-0.11	0.28	0.69	1.16	1.79	3.27	-0.18	0.41	1.03	1.73	2.70	4.93
40 (10%)	$\mathcal{B}$ (M€)	1.74	1.71	1.70	1.70	1.67	1.61	1.73	1.69	1.66	1.64	1.59	1.46	1.71	1.65	1.65	1.60	1.53	1.35
	$\mathcal{B}^*$ (M€)	1.70	1.67	1.65	1.65	1.59	1.47	1.66	1.62	1.57	1.53	1.44	1.19	1.60	1.54	1.52	1.43	1.30	0.94
	DA (M€)	1.74	1.57	1.40	1.21	0.92	0.25	1.74	1.41	1.06	0.67	0.11	-1.26	1.74	1.24	0.75	0.14	-0.71	-2.75
	ID (M€)	-0.004	0.14	0.30	0.49	0.75	1.36	-0.01	0.28	0.60	0.97	1.48	2.72	-0.03	0.41	0.90	1.46	2.24	4.10
	ID* (M€)	-0.04	0.11	0.26	0.43	0.67	1.22	-0.08	0.21	0.52	0.86	1.33	2.45	-0.14	0.30	0.77	1.29	2.01	3.69
92 (25%)	$\mathcal{B}$ (M€)	0.98	0.98	0.98	0.97	0.96	0.94	0.97	0.98	0.97	0.95	0.93	0.86	0.96	0.97	0.97	0.93	0.90	0.84
	$\mathcal{B}^*$ (M€)	0.95	0.95	0.95	0.94	0.91	0.85	0.92	0.93	0.92	0.88	0.83	0.69	0.89	0.89	0.86	0.82	0.75	0.57
	DA (M€)	0.98	0.89	0.79	0.66	0.48	0.07	0.98	0.79	0.58	0.32	-0.03	-0.87	0.98	0.70	0.39	-0.01	-0.53	-1.77
	ID (M€)	-0.004	0.09	0.19	0.31	0.48	0.87	-0.008	0.19	0.39	0.63	0.96	1.74	-0.02	0.27	0.58	0.94	1.43	2.61
	ID* (M€)	-0.03	0.07	0.17	0.28	0.43	0.79	-0.06	0.14	0.34	0.56	0.86	1.56	-0.09	0.20	0.49	0.83	1.28	2.35
316 (50%)	$\mathcal{B}$ (M€)	1.62	1.67	1.62	1.62	1.59	1.47	1.58	1.59	1.57	1.52	1.42	1.17	1.53	1.52	1.48	1.40	1.25	0.90
	$\mathcal{B}^*$ (M€)	1.60	1.65	1.59	1.59	1.55	1.40	1.51	1.55	1.53	1.45	1.34	1.04	1.46	1.46	1.41	1.33	1.13	0.69
	DA (M€)	1.66	1.62	1.48	1.38	1.22	0.79	1.66	1.50	1.30	1.04	0.70	-0.16	1.66	1.39	1.09	0.71	0.17	-1.12
	ID (M€)	-0.04	0.05	0.14	0.24	0.37	0.68	-0.08	0.09	0.27	0.46	0.72	1.33	-0.13	0.13	0.39	0.69	1.08	2.02
	ID* (M€)	-0.06	0.03	0.12	0.21	0.33	0.61	-0.12	0.05	0.23	0.41	0.65	1.19	-0.19	0.07	0.33	0.61	0.97	1.82
262 (75%)	$\mathcal{B}$ (M€)	0.13	0.15	0.16	0.16	0.15	0.11	0.04	0.08	0.10	0.10	0.09	-0.02	-0.05	0.01	0.04	0.05	0.01	-0.17
	$\mathcal{B}^*$ (M€)	0.12	0.14	0.16	0.16	0.15	0.11	0.02	0.06	0.09	0.10	0.08	-0.02	-0.08	-0.01	0.03	0.04	0.01	-0.18
	DA (M€)	0.22	0.20	0.19	0.17	0.15	0.08	0.22	0.19	0.16	0.12	0.07	-0.06	0.22	0.17	0.13	0.07	-0.01	-0.24
	ID (M€)	-0.09	-0.06	-0.03	-0.01	0.01	0.02	-0.18	-0.11	-0.06	-0.02	0.01	0.05	-0.27	-0.16	-0.09	-0.03	0.02	0.07
	ID* (M€)	-0.10	-0.06	-0.03	-0.01	0.01	0.02	-0.20	-0.13	-0.07	-0.02	0.01	0.04	-0.30	-0.18	-0.10	-0.03	0.02	0.06
105 (90%)	$\mathcal{B}$ (M€)	0.12	0.12	0.11	0.11	0.11	0.08	0.09	0.09	0.09	0.07	0.04	-0.08	0.05	0.06	0.04	0.03	-0.04	-0.23
	$\mathcal{B}^*$ (M€)	0.11	0.11	0.09	0.09	0.08	0.03	0.06	0.06	0.05	0.03	-0.01	-0.16	0.01	0.02	0.00	-0.03	-0.11	-0.35
	DA (M€)	0.15	0.10	0.03	-0.02	-0.11	-0.32	0.15	0.05	-0.06	-0.20	-0.38	-0.85	0.15	0.00	-0.18	-0.37	-0.67	-1.40
	ID (M€)	-0.03	0.02	0.08	0.14	0.22	0.40	-0.06	0.04	0.15	0.27	0.42	0.78	-0.10	0.06	0.22	0.40	0.63	1.17
	ID* (M€)	-0.04	0.01	0.06	0.12	0.19	0.36	-0.09	0.01	0.12	0.23	0.38	0.70	-0.14	0.02	0.17	0.34	0.56	1.05
295 (99%)	$\mathcal{B}$ (M€)	0.38	0.39	0.40	0.39	0.39	0.38	0.34	0.36	0.37	0.37	0.35	0.30	0.30	0.36	0.33	0.33	0.31	0.20
	$\mathcal{B}^*$ (M€)	0.37	0.38	0.38	0.38	0.38	0.35	0.32	0.33	0.34	0.33	0.32	0.25	0.26	0.28	0.29	0.29	0.26	0.12
	DA (M€)	0.42	0.38	0.35	0.30	0.24	0.10	0.42	0.35	0.28	0.19	0.06	-0.24	0.42	0.32	0.20	0.07	-0.12	-0.61
	ID (M€)	-0.04	0.005	0.05	0.09	0.15	0.28	-0.08	0.005	0.09	0.17	0.29	0.54	-0.12	0.04	0.13	0.26	0.43	0.81
	ID* (M€)	-0.05	-0.01	0.03	0.08	0.13	0.25	-0.10	-0.02	0.06	0.15	0.26	0.49	-0.15	-0.03	0.09	0.22	0.38	0.73

Last, the attainable savings for consumers are modest in all cases. At most, a consumer may be able to reduce its energy bill by 2.64 €/day. However, such savings are only attainable on days with a high heating load. During the rest of the heating season, savings may be limited to, on average, 0.72 €/day (IDM price mark-ups, $\sigma^{\mathrm{P}} = 0.15$, $\sigma^{\mathrm{NP}} = 150$ MW) to 0.84 €/day (no IDM price mark-ups, $\sigma^{\mathrm{P}} = 0.05$, $\sigma^{\mathrm{NP}} = 50$ MW). These savings further decrease and may even turn negative if the aggregator is unable to select an optimal risk attitude ϵ. In the last case, the aggregator may need to absorb the entire monetary loss to keep consumers engaged.

In summary, the aggregator must find an optimal risk-attitude ϵ, i.e., an optimal balance between the volume procured in the DAM and the IDM, in order to secure the maximal benefit $\mathcal{B}$. Remarkably, although a less liquid IDM reduces the overall attainable benefit $\mathcal{B}^*$, the highest benefits are observed for similar risk-attitudes ϵ, regardless of the liquidity of the IDM. However, reduced liquidity of the IDM increases the importance of selecting an optimal risk-attitude, since benefit $\mathcal{B}^*$ decreases more strongly for non-optimal ϵ-values than the attainable benefit $\mathcal{B}$ considering a liquid IDM.

C. Computational effort

The aggregator's day-ahead decision problem, considering the Nash Bargaining paradigm, contains 318,241 constraints (of which 96 are quadratic constraints) and 176,160 variables (of which 107,040 are continuous and 69,120 binary). All models are implemented in GAMS v24.4 and solved using Gurobi v8.0. The stopping tolerance was set to 1%. Simulations are performed using a 2.8GHz machine with 4 cores and 8GB of RAM. On average, an instance is solved in 770 seconds (median value: 497 seconds). The minimum computation time we recorded was 60 seconds, whereas the solver timed out for a single instance after 3,600 seconds. In all cases, however, the observed MIP gap is below 1%.

IV. Conclusion

Demand response (DR) resources, such as thermostatically controlled loads, may provide the required flexibility to compensate from variations and unexpected deviations in, e.g., the expected electricity generated from renewable energy sources and other loads. Aggregators may be needed to bring these

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distributed, low-capacity resources to day-ahead (DAM) and intra-day electricity markets (IDM).

We study the interaction between (i) an aggregator and a DAM via a Stackelberg game and (ii) an aggregator and its DR providers via a Stackelberg and a Nash Bargaining Game. To manage the limited controllability of DR loads, we propose chance constraints. The uncertainty in the aggregator's anticipation on the DAM outcome is represented via scenarios. For the specific case of DR with electric heating systems and guaranteed thermal comfort, we show that, assuming DR providers without bargaining power, the outcome of a Stackelberg Game between the aggregator and its consumers, if it exists, is a subset of the set of possible outcomes of a Nash Bargaining Game, which allows representing the aggregator and DR providers as a single entity participating in the DAM.

In a case study, inspired by the Belgian power system, we illustrate how the aggregator may lower day-ahead prices and how it may manage limitedly controllable resources. As the DR resource becomes less controllable and the aggregator becomes more risk-averse, the attainable profits in the DAM decrease, which may not be fully compensated in the IDM. Nevertheless, assuming a well-functioning and liquid IDM, an aggregator is able to attain an operating profit w.r.t. a retailer throughout the heating season. Only when the limited controllability of the DR resource increases and the DR aggregator becomes strongly risk-averse, operating profits may turn negative. A reduced liquidity of the IDM reduces the attainable benefits and increases the importance of selecting an optimal risk-attitude. The attainable benefits for consumers may be limited, which may threaten the viability of the aggregator-consumer interaction model.

The presented methodology may be used by regulators, policy makers and power system operators to assess the value of DR in a deregulated power system or may be directly integrated in the daily routine of DR aggregators.

Our research may be strengthened by introducing multiple segments of consumers, each with a specific bargaining power. Second, alternative assumptions on the capabilities of the DR aggregator to adapt its position in the IDM could change the results above. Similarly, explicitly representing the IDM in the aggregator's day-ahead bid problem may lead to more efficient bid strategies. Third, the interaction between (1) limited controllability of DR loads, (2) IDMs and (3) operating reserve requirements and balancing markets may be a subject worthy of investigation. Fourth, our work would benefit from an analysis of how, e.g., ownership structures affect the bargaining power and hence the outcomes of the Nash Bargaining Game. Last, introducing transmission and distribution grid constraints may lead to interesting insights in the location-specific value of DR resources and the coordination of these resources to meet flexibility needs in the distribution and transmission grid.

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